A Grammar Compression Algorithm based on Induced Suffix Sorting

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Summary

1. Introduction
2. GCIS
3. Results
4. Final Considerations

Summary

1. Introduction

Introduction

- Lossless data compression reduce space requirement by identifying and eliminating redundancy.
- Useful in practice: reduce resources required to store and transmit data.
- Space/Time trade-off.

Introduction

- The Suffix Array Data Structure is used extensively in Stringology.
- Key to solve several text-related problems in efficient or optimal time, using a small footprint of memory, if compared to other data structures.
  - Exact Pattern Matching;
  - Approximate Pattern Matching;
  - LZ factorization;
  - Finding repeats.

Suffix Array

i	$A[i]$	$T_{A[i]}$
0	22	0
1	21	a0
2	18	aana0
3	13	aananaana0
4	8	aananaananaana0
5	3	aananaananaananaana0
6	19	ana0
7	16	anaana0
8	11	anaananaana0
9	6	anaananaananaana0
10	1	anaananaananaananaana0
11	14	ananaana0
12	9	ananaananaana0
13	4	ananaananaananaana0
14	0	banaananaananaananaana0
15	20	na0
16	17	naana0
17	12	naananaana0
18	7	naananaananaana0
19	2	naananaananaananaana0
20	15	nanaana0
21	10	nanaananaana0
22	5	nanaananaananaana0

Nong's Algorithm

- Nong *et al.* algorithm is capable of sorting the suffixes of a text in linear optimal time.
- Very fast in practice.
- Induces the order of suffixes based in the already calculated order of other suffixes.

Our Contribution

- Develop a novel grammar compression algorithm based on the induced suffix sorting algorithm from Nong *et al.* to compress the original string.
  - Faster in compression than 7-ZIP and RE-PAIR.
  - Lower memory peak under compression than RE-PAIR.

Summary

2 GCIS

GCIS

We adapt SAIS framework from Nong *et al.* to build our grammar:

1. Classify suffixes into "S", "L", or "LMS" types.
2. Sort all LMS-suffixes by its first symbol.
3. Induce:
   1. L-type suffixes from LMS-type suffixes;
   2. S-Type suffixes from L-type suffixes;
4. Rename the LMS-Substrings to obtain T' and create grammar rules;
5. If there are equal renamed factors, solve the problem recursively for T'. Else, store T explicitly.
6. LMS-suffixes are now sorted regarding all symbols. Repeat 3.1 and 3.2 to obtain the suffix array.
7. ~~LMS-suffixes are now sorted regarding all symbols. Repeat 3.1 and 3.2 to obtain the suffix array.~~

GCIS

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
T	b	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	0

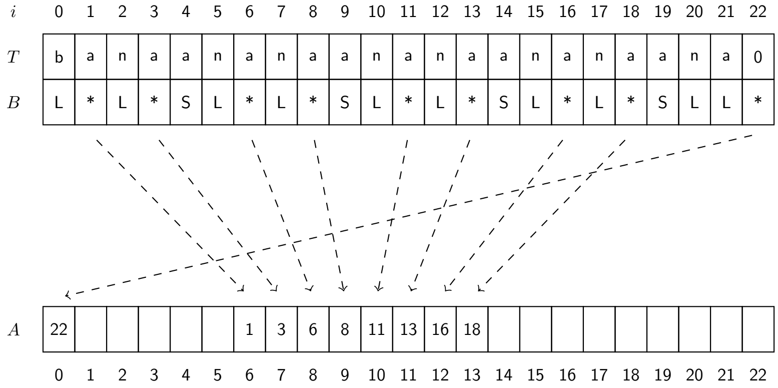
GCIS

Classification

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
T	b	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	0
B	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	L	*

GCIS

Radixsort

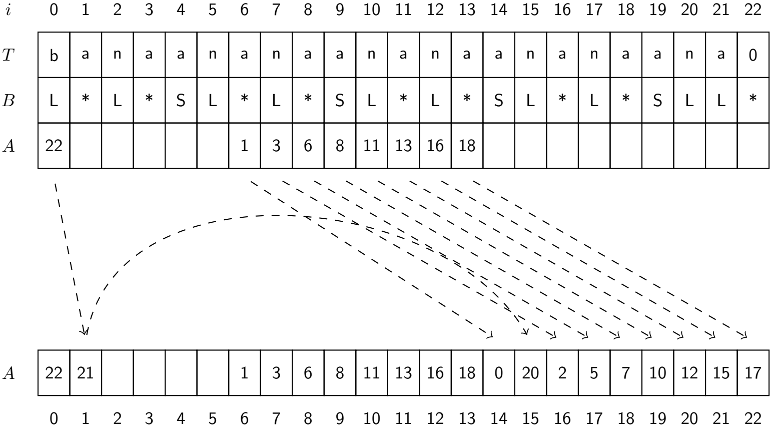


i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
T	b	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	0
B	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	L	*

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	22						1	3	6	8	11	13	16	18									

GCIS

Inducing L-Type Suffixes

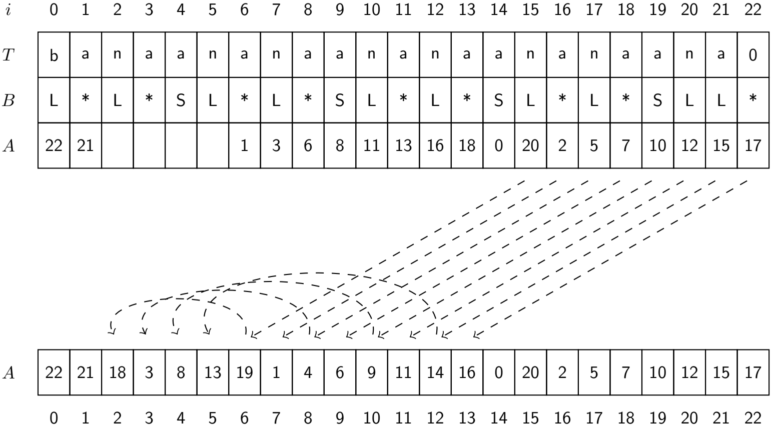


i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
T	b	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	0
B	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	L	*
A	22						1	3	6	8	11	13	16	18									

A	22	21					1	3	6	8	11	13	16	18	0	20	2	5	7	10	12	15	17
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

GCIS

Inducing S-Type Suffixes



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
T	b	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	0
B	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	L	*
A	22	21					1	3	6	8	11	13	16	18	0	20	2	5	7	10	12	15	17

A	22	21	18	3	8	13	19	1	4	6	9	11	14	16	0	20	2	5	7	10	12	15	17
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

GCIS

Definition (LMS-Substring)

A LMS-Substring is either the sentinel symbol 0 or a substring $T[i, j-1]$ with both $T[i]$ and $T[j]$ being of type LMS and there is no other LMS symbols for $i \neq j$. We denote this substring by $sub(i)$.

- After inducing L and S-type suffixes, all LMS-substrings are sorted.
- All LMS-substrings are renamed according to their order.
- A pairwise comparison should be done to check if the LMS-substrings are equal.
- T is replaced by the renamed factors, giving place to T'.

GCIS

Renaming

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
T	b	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	n	a	a	n	a	0
B	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	*	L	*	S	L	L	*
A	22	21	18	3	8	13	19	1	4	6	9	11	14	16	0	20	2	5	7	10	12	15	17
	*		*	*	*	*		*		*		*		*									

$$\begin{array}{ll} sub(22) = 0 \mapsto 0 & sub(1) = an \mapsto 3 \\ sub(18) = aana \mapsto 1 & sub(6) = an \mapsto 3 \\ sub(3) = aan \mapsto 2 & sub(11) = an \mapsto 3 \\ sub(8) = aan \mapsto 2 & sub(16) = an \mapsto 3 \\ sub(13) = aan \mapsto 2 & \end{array}$$

$$T' = 323232310$$

Creating Rules

- For every unique LMS-substring S a rule of the form $X \to S$ is created.
- In order to obtain a good compression ratio, certain properties shared between the LMS-substrings shall be explored.
- The first property is that the LMS-substrings are sorted!
- We can represent them by a pair (ℓ, s):
    - ℓ: the common prefix length shared with the previous LMS-substring.
    - s the remaining suffix not encoded by ℓ.
- A special rule is created to store the first symbols of T not covered by a LMS-substring.

Encoding the Factors

For $T = banaananaananaananaana0$

- $0 \rightarrow (0,\text{'}0\text{'})$; //'0';
- $1 \rightarrow (0,\text{'}aana\text{'})$; // 'aana'
- $2 \rightarrow (3,\text{''})$; // 'aan'
- $3 \rightarrow (1,\text{'}n\text{'})$; // 'an'
- $Tail \rightarrow b$

Encoding the Factors

- The ℓ values can be encoded succinctly by using `Simple-8b`
- Tries to fit as many fixed-width integer as possible in a 64-bit word.
- With a single memory access, many values are retrieved.

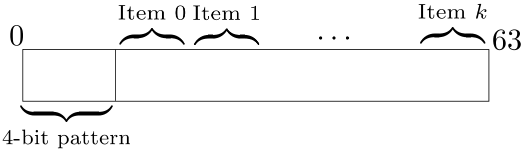


Table: `Simple8b` possible arrangements.

Selector value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Item width	0	0	1	2	3	4	5	6	7	8	10	12	15	20	30	60
Group Size	240	120	60	30	20	15	12	10	8	7	6	5	4	3	2	1
Wasted bits	60	60	0	0	0	0	0	0	4	4	0	0	0	0	0	0

Encoding the Factors

- The s parts are concatenated in a single array of integers with fixed width of $\lceil \lg(|\Sigma|) \rceil$ bits.

$$V = 0aanan$$

- A support bitmap encoding the length of each s part is created.

$$BV = 0100001101$$

- Length of suffix $i = \textsc{select}_1(BV, i+1) - \textsc{select}_1(BV, i) - 1$
- Start of suffix i in V $= \textsc{select}_1(BV, i) - i + 1$.
- To improve space usage, the bitmap is encoded using Elias-Fano.

Decoding

- We begin by the reduced string.
- At each recursion level, we read $T[i] = X$ and replace it by its the correct LMS-substring $X \rightarrow S$.
- To make the decoding faster, all rules from the current level are decompressed previously.

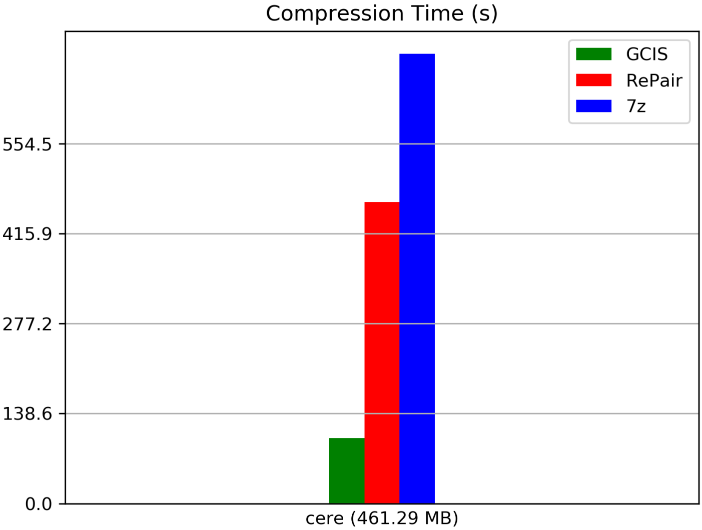
Summary

3. Results

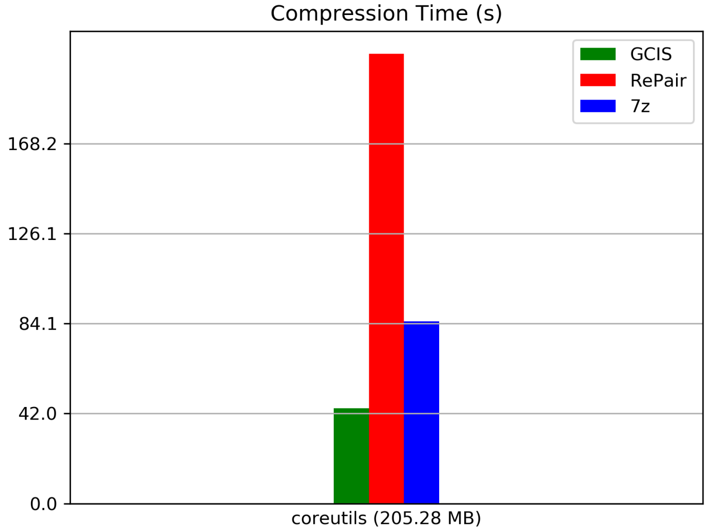
Results

- In order to visualize the potential of the proposed grammar compressor (GCIS), experiments were done with the repetitive Pizza-Chili *corpus*, which was populated with texts of different nature.
- GCIS was compared against popular compressors suited to repetitive sequences: RE-PAIR and 7-ZIP.
- Three subjects were evaluated:
  - Compression ratio: compressedSize/Size.
  - Compression time (s).
  - Decompression time (s).

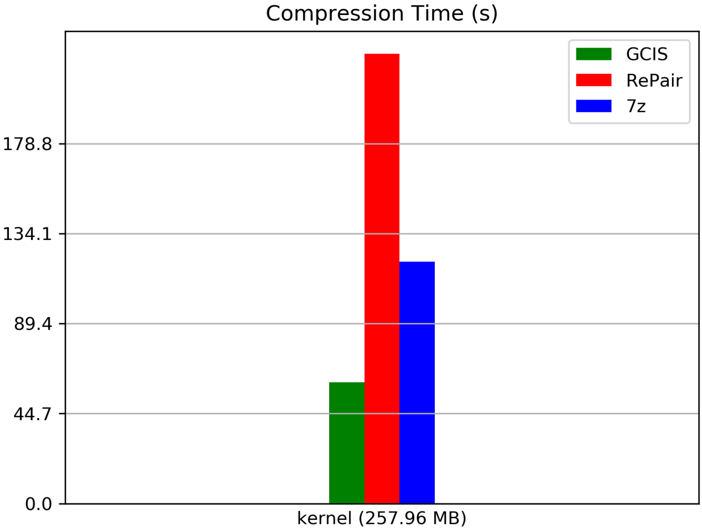
Compression Time



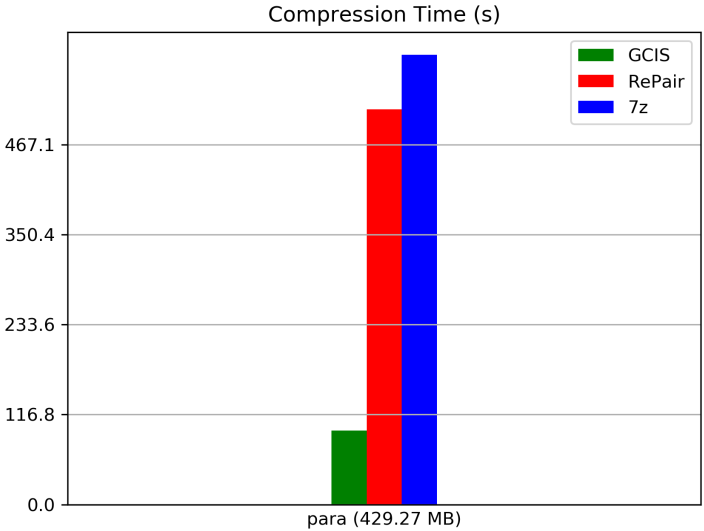
Compression Time



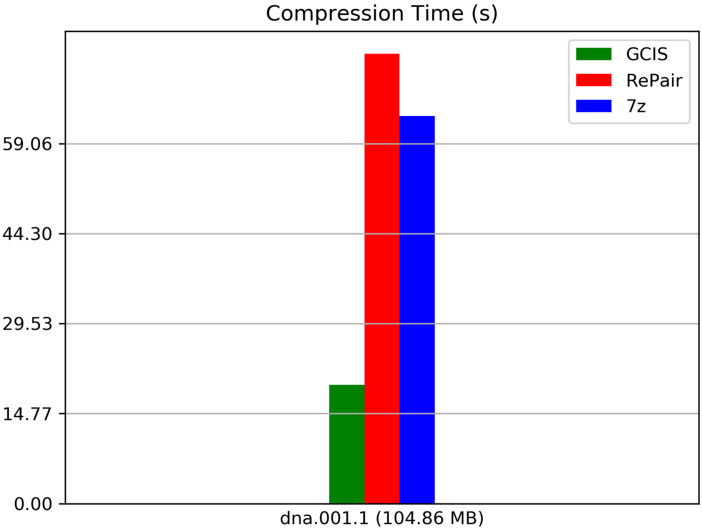
Compression Time



Compression Time



Compression Time



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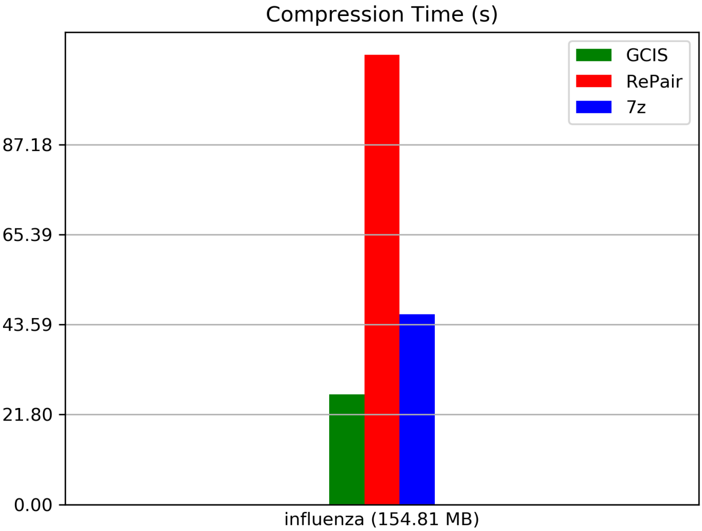

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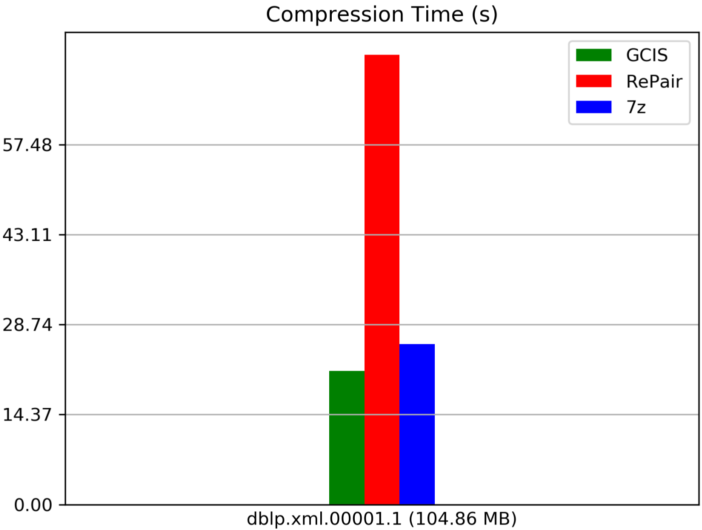
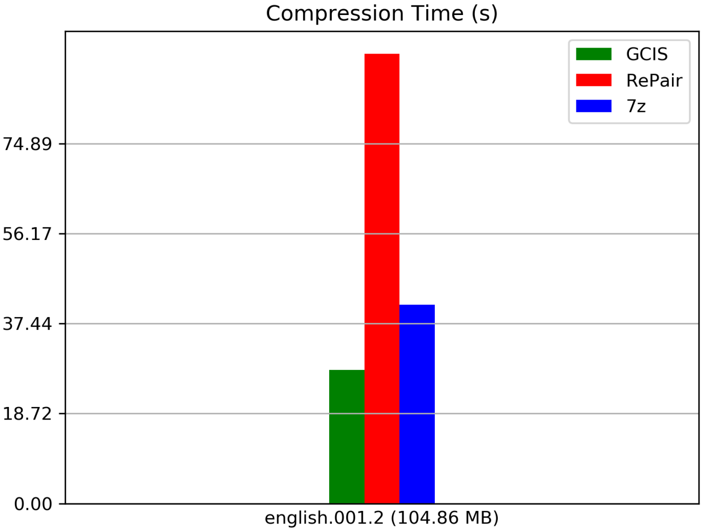
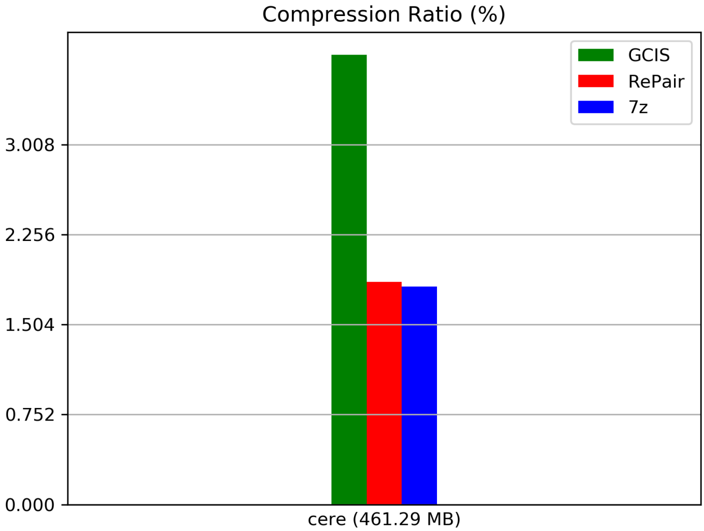


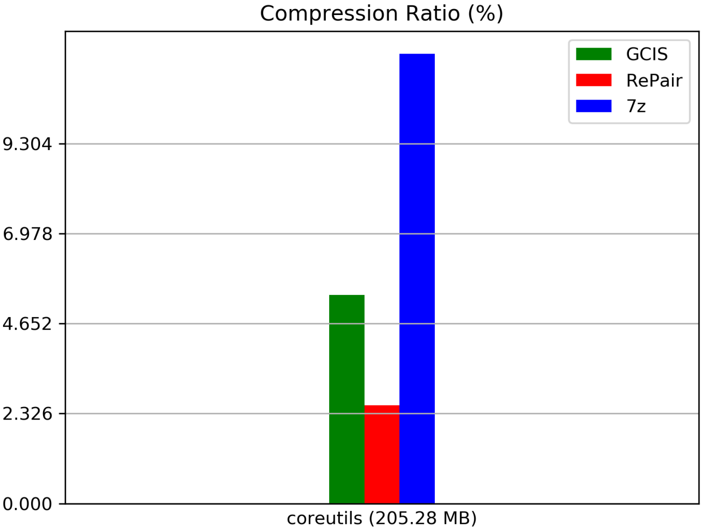
Compression Time



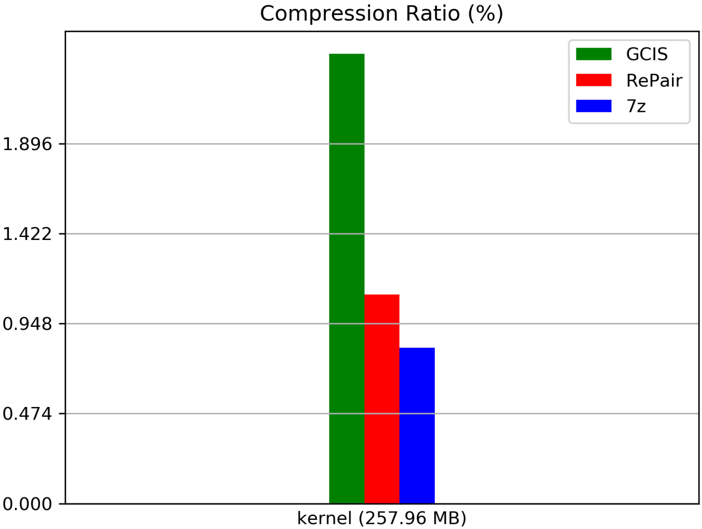
Compression Ratio



Compression Ratio



Compression Ratio



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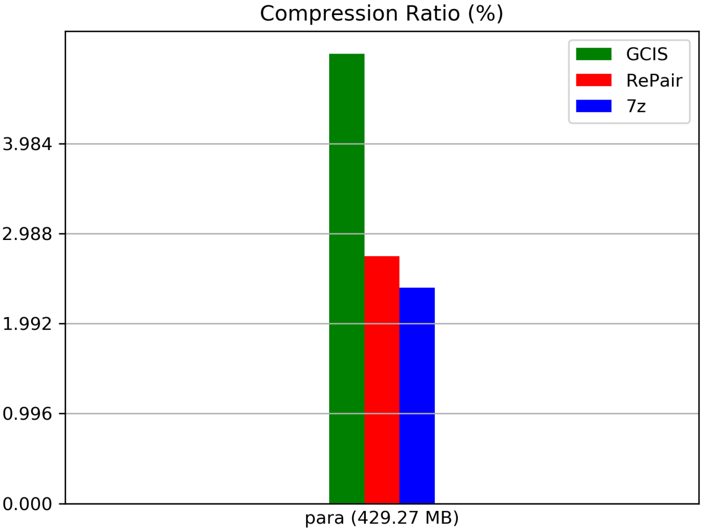

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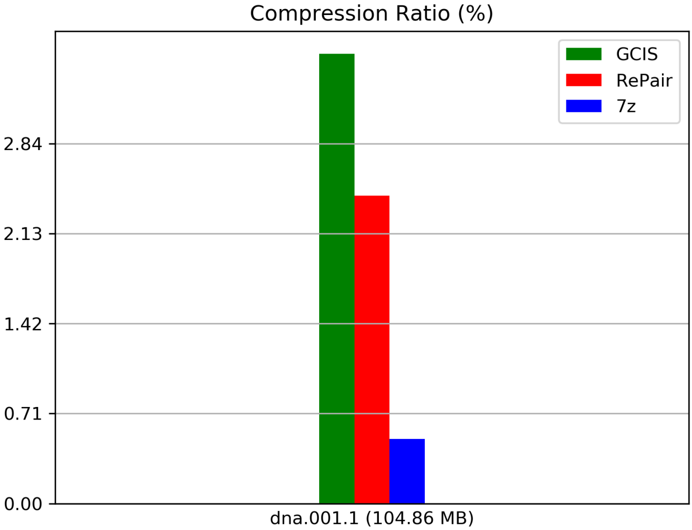

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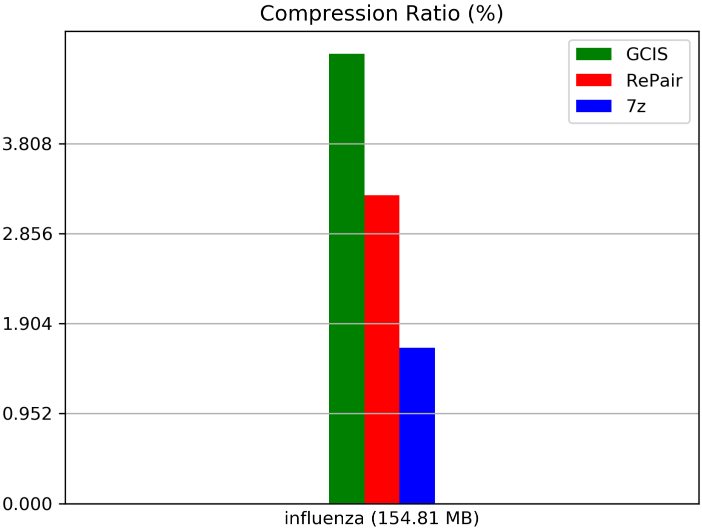

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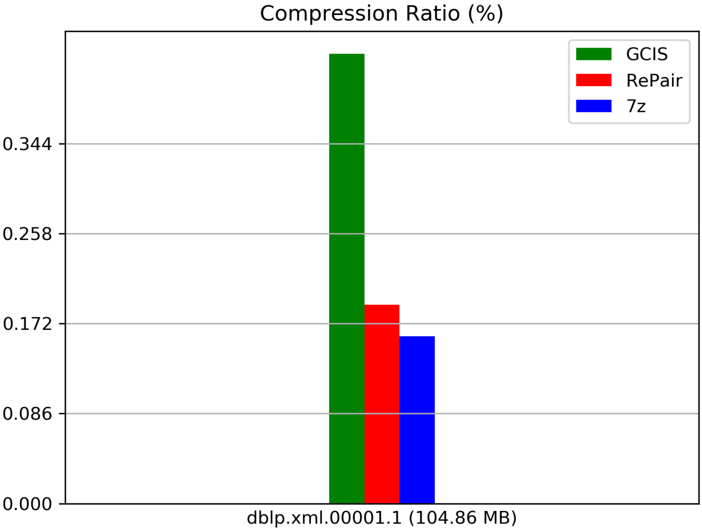
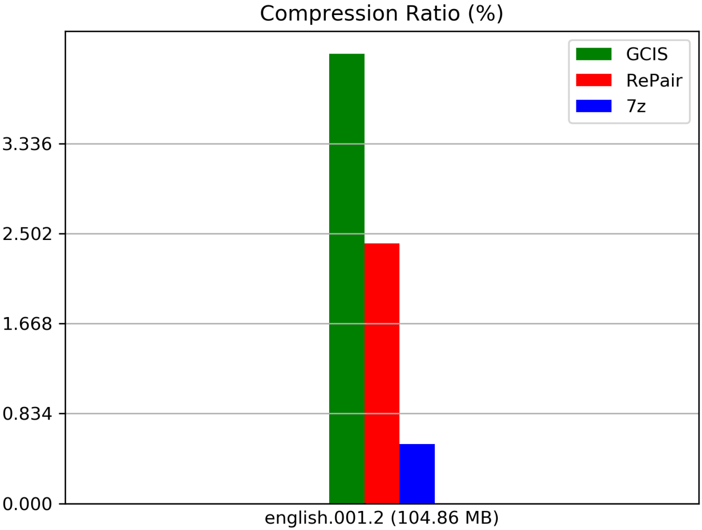
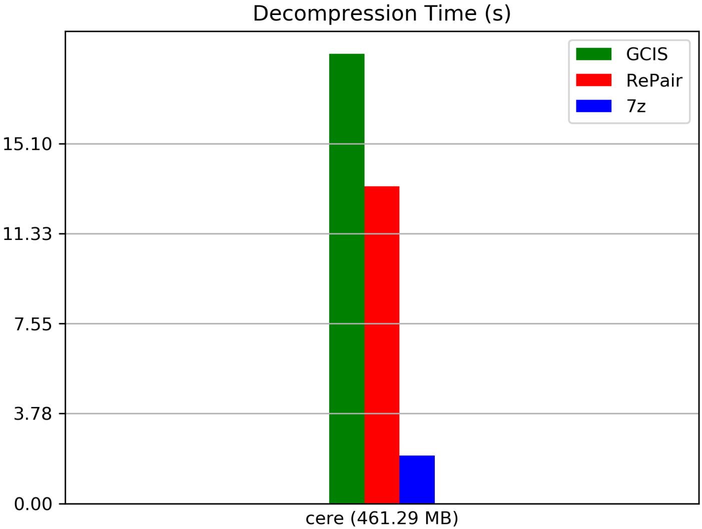


Compression Ratio



Decompression Time



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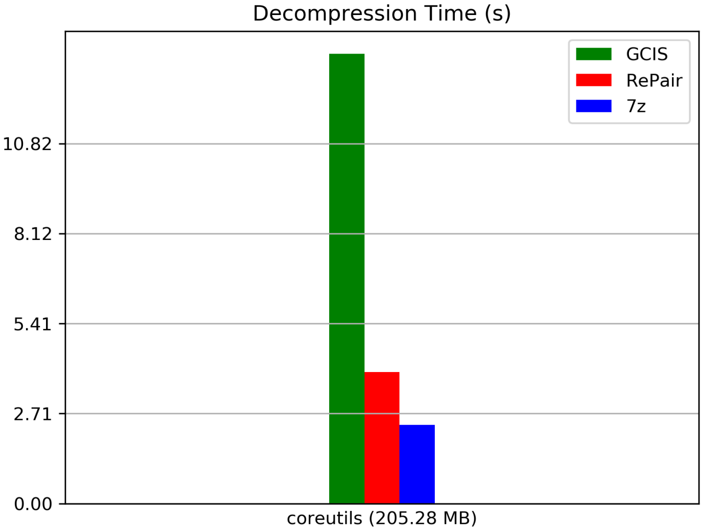
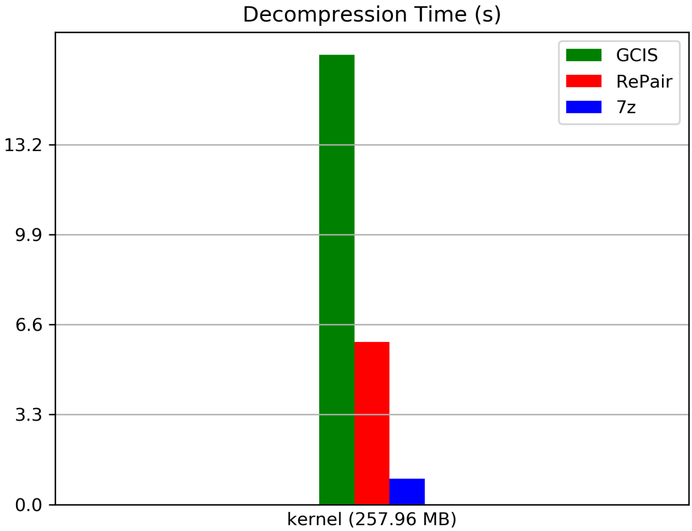
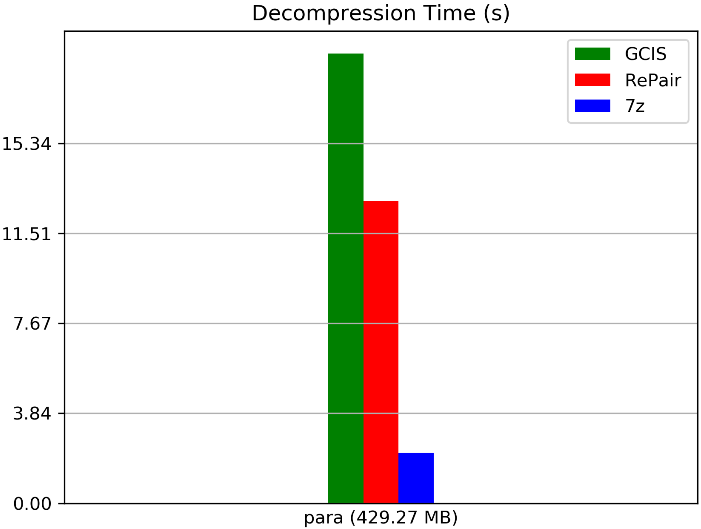


Decompression Time



Decompression Time



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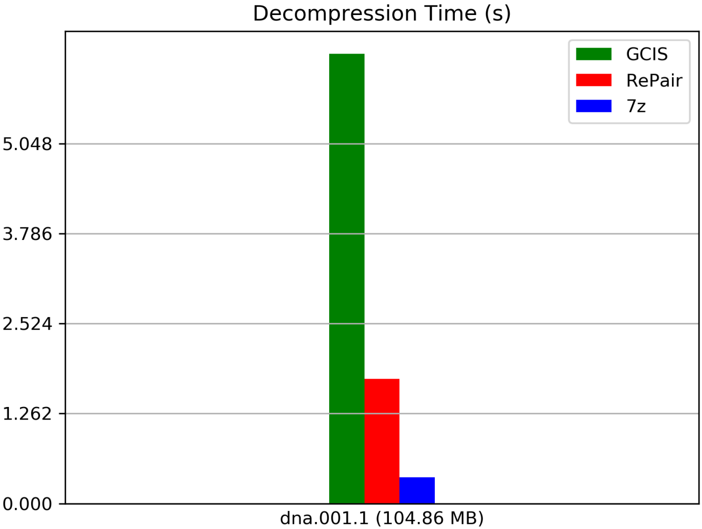

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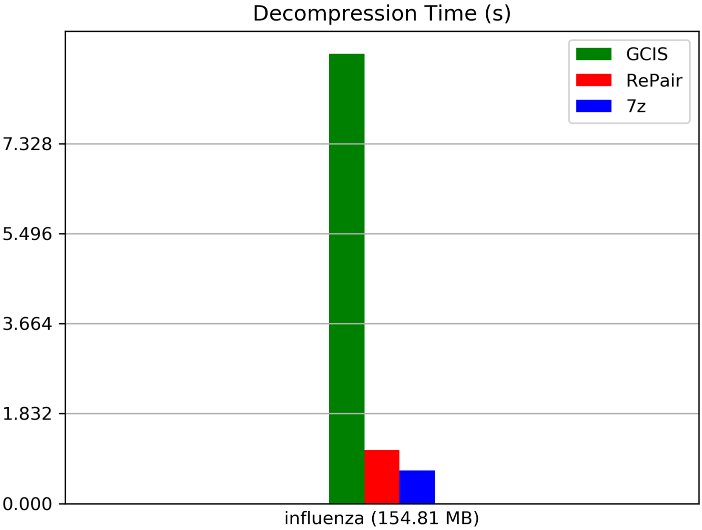

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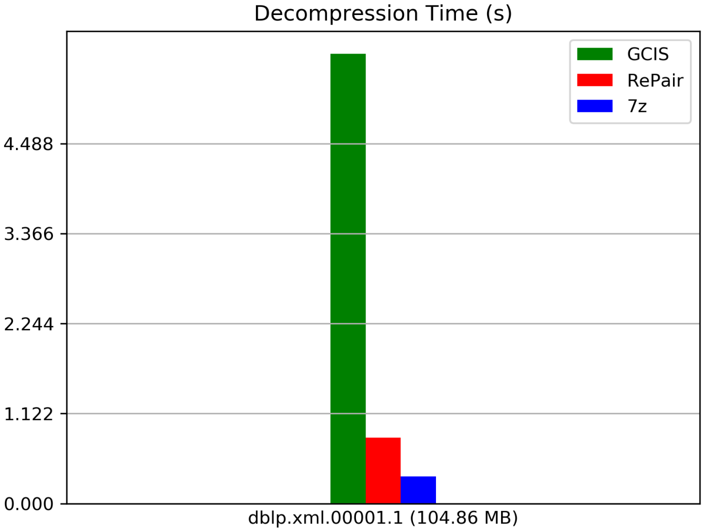
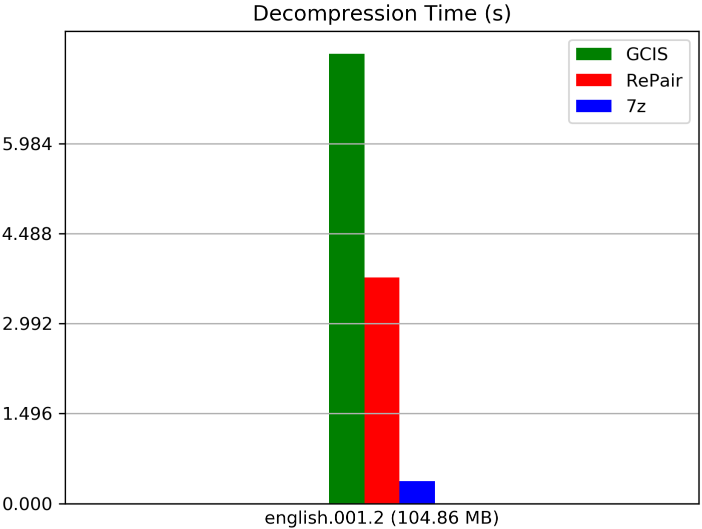
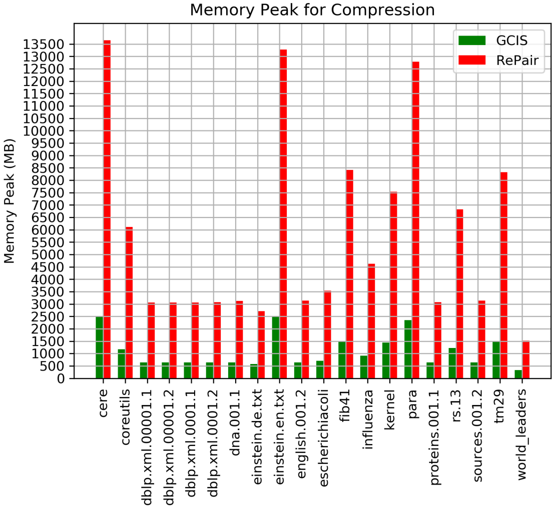


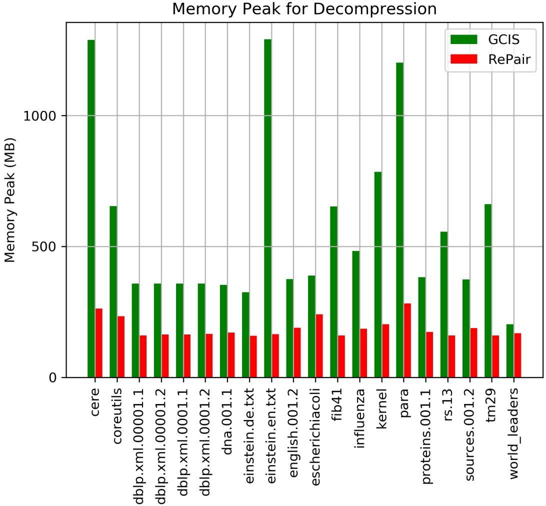
Decompression Time



Peak Memory during Compression



Peak Memory during Decompression



Summary

4 Final Considerations

Final Considerations

- GCIS showed to be a practical alternative to popular compressors due its good compression ratio and time.
- Competitive in compression regarding 7-ZIP and RE-PAIR and much faster than both.
- Slower decode times.
- Worse compression ratio, but comparable to RE-PAIR.
- Lower memory peak than RE-PAIR.

Work in Progress

- It is possible to support extraction of any substring without much space by storing the rule lenghts succinctly.
- RE-PAIR also supports EXTRACT but in a less space-efficient version which requires $(2.x$ to 3 times more space$)$.
- 7-ZIP does not support extraction.
- Develop extraction and compare with the less space-efficient version of RE-PAIR.
- We hope that GCIS will be more space-efficient than RE-PAIR when supporting the EXTRACT operation.

Thank you

Thank you!